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COMPRESSION TESTS OF SOME 17S-T ALUMINUM-ALLOY SPECIMENS OF I CROSS SECTION

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SUMMARY

Specimens cut from a specially extruded I-beam of 17S-T aluminum alloy with one flange wider than the other were tested under axial compression. The lengths of the specimens varied from 4 to 90 inches. Only the two longer specimens failed as columns; the shorter ones failed by local buckling.

The results, when compared with theoretical values, showed that the design methods proposed by the Aluminum Company of America are satisfactory.

INTRODUCTION

The strength of a specimen or a member subjected to an axial compressive force will be governed by some form of instability unless the specimen is extremely short or thick. Extremely short pieces, such as block compression specimens, may fail by excessive plastic distortion unaccompanied by buckling. Instability in a specimen under axial compressive load may result in a primary buckling failure in which the various cross sections are displaced relative to each other but in which there is no distortion in the shape of any cross section. This type of failure includes the lateral and the torsional buckling of columns. On the other hand, instability may be of a part of the cross section rather than of the member as a whole, in which case failure occurs by local buckling accompanied by distortion of the shape of the cross section. Methods are given in reference 1 for treating both types of failure in the design of aluminum-alloy structures. These design methods are based on rational theoretical analyses of the problems involved and, in most cases, are substantiated by experience and the results of laboratory tests.

It is always well, however, to consider current design methods in the light of newly acquired experimental evidence.

The test results discussed in this report were obtained as a portion of a general investigation outlined for the purpose of studying the compression buckling of various structural elements. The investigation included beam tests as well as end compression tests. The results of the beam tests have been discussed in reference 2.

The purpose of the present paper is to discuss some axial compressive tests made on 17S-T specimens of I cross section, with emphasis on a comparison of the test results with current design methods.

DESCRIPTION OF SPECIMENS

The specimens were cut from lengths of a specially extruded 17S-T I-beam having one flange wider than the other flange. Table I gives the dimensions of the various specimens tested. The dimensions shown in the diagram of table I are the average of the measured dimensions for all specimens in the group. With but one or two exceptions, the maximum variation of any individual measurement from the average value was about 1 percent.

The ends of all specimens were carefully machined so as to be flat, parallel, and normal to the axis of the specimen.

Methods of Testing

All the specimens were tested between fixed heads in the 40,000-pound capacity Amsler testing machine. Lateral deflections were measured at the middle of each specimen, the wire-and-mirror-scale method being used and the mirror scale being fastened to the narrow flange.

Tensile tests were made on specimens cut longitudinally from the web and the flange of each extruded length. Since the investigation involved the behavior of specimens under compressive loads, knowledge of the compressive stress-strain relations of the material would be desirable.

Such information could not be obtained from compression tests on short lengths of the extruded section because of local buckling of the web and the flanges at stresses below the yield strength of the material. The pack method (reference 3) for testing thin-sheet material in compression did not become available until some time after the completion of the investigation. A sample of only one extruded length was then available (length marked V, table II). A pack compression test was made on material cut longitudinally from the web of this sample.

Results of Tests

The tensile properties of the material in the various extruded lengths are summarized in table II. The results of the pack compressive test on the material from the web of the extruded length marked V, have been plotted in figure 1 as a stress-strain curve. The compressive yield strength for this material (39,500 lb per sq in.) was somewhat lower than the tensile yield strength of 45,000 pounds per square inch given in table II. This difference between tensile and compressive yield strengths probably results from stretching the extruded lengths to straighten them.

TABLE II. - TENSILE PROPERTIES OF MATERIAL

[Specimens cut in longitudinal direction]

Length marked	Specimens cut from web			Specimens cut from flange		
	Tensile strength (lb/sq in.)	Yield strength[a] (lb/sq in.)	Percentage of elongation in 2 inches	Tensile strength (lb/sq in.)	Yield strength[a] (lb/sq in.)	Percentage of elongation in 2 inches
V	57,690	45,000	18.5	55,320	42,600	21.0
W	58,030	42,400	21.5	58,090	41,300	23.0

[a]Stress at 0.2 percent set.

TABLE III. - ULTIMATE COMPRESSIVE STRENGTH OF 17S-T

SPECIMENS OF I CROSS SECTION

Specimen	L/r	Ultimate load, P (lb)	Ultimate stress[a], P/A (lb/sq in.)
1	194.5	10,175	9,250
2	129.7	21,250	19,130
3	78.0	31,000	28,180
4	52.1	35,680	32,120
5	26.1	36,300	32,560
6	17.5	36,300	33,150
7	8.88	38,470	34,720

[a]Because of the proportions of the cross section, the ultimate stress practically coincided with the critical stress for the web of the I, for specimens that failed by local buckling.

The results of the tests are summarized in table III. Only specimens 1 and 2, which failed primarily as columns, buckled at average stresses within the elastic range of the material. Specimens 3 to 7 failed by local buckling of the web and the flange. A load-lateral deflection curve is shown in figure 2 for only specimen 2. The other specimens of this group failed suddenly with no appreciable previous lateral deflections having been measured. No lateral-deflection data were obtained for specimens 3 to 7 because the deflections were measured at the middle of the specimen relative to its ends, whereas failure occurred by buckling of a short length of the web and the flanges. Not enough lateral-deflection measurements were taken of specimen 1 to establish a load-deflection curve.

Analysis of Test Results

Only two of the specimens (specimens 1 and 2) failed primarily as columns; the other five failed by local buckling. The end restraint of a column with flat ends, tested between fixed heads, is nearly as great as if the ends were completely fixed. The critical stress for an axially loaded column that buckles by bending within the elastic range of the material may be expressed

$$\frac{P'}{A} = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} \qquad (1)$$

where P' critical load, pounds

A cross-sectional area, square inches

E modulus of elasticity, pounds per square inch

L length of column, inches

r least radius of gyration of column, inches

k coefficient defining the degree of end restraint (for round ends, $k = 1$; for fixed ends, $k = \frac{1}{2}$)

A substitution into equation (1) of the experimentally determined values of P' for specimens 1 and 2 (table II) yields values for the fixity coefficient, k, of 0.54 for specimen 1 and 0.56 for specimen 2. The column strength of any of the specimens, within the elastic range, may then be calculated by equation (1) with $k = 0.55$. If buckling occurs within the plastic range of the material, E in equation (1) is no longer Young's modulus but is a reduced value depending on the value of the average stress and to some extent on the shape of the cross section. (See reference 4, p. 156.) From numerous tests on aluminum-alloy columns of various cross sections it has been determined that the behavior within the plastic range can be approximated by the linear relation between critical stress and slenderness ratio represented by a straight line tangent to the Euler curve and intersecting the stress axis at a value determined from the equation (see reference 5)

$$B = CYS\left(1 + \frac{CYS}{200000}\right) \qquad (2)$$

where CYS is the compressive yield strength of the material.

The formulas given on page 37 of reference 1 for determining the strength of columns of various aluminum alloys are based on this relationship. In these formulas the critical stress is expressed in terms of the "effective" slenderness ratio, which is the slenderness ratio, L/r, multiplied by the fixity coefficient, k. The Handbook formula for the strength of short columns of 17S-T, based on a typical value of the compressive yield strength of the material, CYS, of 37,000 pounds per square inch, is

$$\frac{P}{A} = 43{,}800 - 350\,\frac{kL}{r} \tag{3}$$

The column curves based on this equation have been plotted in figure 3, which also shows the ultimate strength of the I-shaped specimens tested. When these test results were plotted, the slenderness ratio, L/r, for each specimen was multiplied by 0.55, which is the k value determined from tests of specimens 1 and 2.

It will be noticed in figure 3 that the experimental points for the short specimens fall below the column curve. This lack of agreement may be explained by the fact that failure in these specimens occurred by local buckling rather than primarily as a column. The cross section of these specimens may be considered as composed of thin rectangles. The critical stress for a flat rectangular plate subjected to uniform edge compression in one direction can be expressed

$$\sigma_{cr} = K\,\frac{E}{1-\mu^2}\left(\frac{t}{b}\right)^2 \tag{4}$$

where σ_{cr} critical stress, pounds per square inch

E modulus of elasticity, pounds per square inch

μ Poisson's ratio

t thickness of plate, inches

b width of plate (normal to direction of stress), inches

K coefficient depending on ratio of length to width (L/b) of plate, the nature of restraint at edges of plate, and in some cases on the Poisson's ratio of the material (reference 6)

If buckling occurs within the plastic range of the material, E is not Young's modulus but is a reduced value depending upon the average stress in the plate. In reference 1 the buckling of flat plates is handled by expressing the variables involved in determining the critical stress in terms of an equivalent slenderness ratio and then determining the critical stress from a column curve. Such an expression for equivalent slenderness ratio can be obtained by equating P/A in equation (1) and σ_{cr} in equation (4). The equivalent slenderness ratio may then be expressed

$$\frac{kL}{r} = \left(\frac{b}{t}\right) \frac{\pi}{\sqrt{K}} \sqrt{1 - \mu^2} \qquad (5)$$

A comparison of the relative stability of the web and the wide flange of the I-section indicates that local buckling would first occur in the web. If it is assumed that the juncture between the web and the flange represents a supported edge, the formulas given on page 41 of reference 1 are for the equivalent slenderness ratio of the web

$$\frac{kL}{r} = 1.65 \frac{b}{t}$$

and for the outstanding part of the flange

$$\frac{kL}{r} = 5.1 \frac{b}{t}$$

Inasmuch as the b/t for the web is 33.1 and for the wide flange is 9.3, the equivalent slenderness ratio values are 54.6 and 47.4, respectively, for the web and the flange. The element with the greater equivalent slenderness ratio will buckle at the lower stress. Since there is so little difference in strength between the web and the flanges, buckling of the web will be accompanied by a redistribution of stress that will result in buckling of the flanges. Consequently, complete failure may be expected at a load very little greater than that at which the web buckled. This result is particularly true if the buckling stress for the web is in the plastic range of the material, as was the case in these tests. For practical purposes, the ultimate load may then be considered as the load that produced buckling of the web. The condition of restraint at the edges of the web lies somewhere between supported and fixed. For the case in which the unloaded edges are built in (fixed), the Handbook gives for the equivalent slenderness ratio (reference 1, p. 41)

$$\frac{kL}{r} = 1.25 \frac{b}{t}$$

Since the ratio, b/t, is 33.1, the equivalent slenderness ratio for this case is 41.4.

In figure 3 are drawn horizontal lines that intersect the column curve at points corresponding to kL/r values of 41.4 and 54.6. These lines represent the critical stress for the method of calculation given in reference 1 for the extreme conditions of edge restraint. The lines are horizontal because the formulas given in the Handbook assume that the critical stress is independent of the length of the member.

It will be noticed in figure 3 that the test results gave higher values for local buckling than those represented by either horizontal line. This discrepancy can be partly accounted for by the fact that the column curve of figure 3 is based on a typical value of the compressive yield strength of the material, CYS, of 37,000 pounds per square inch; whereas, the yield strength determined from a pack compression test on the material from the web was 39,500 pounds per square inch (fig. 1). No compressive stress-strain curve for the material from length W being available, it will be assumed that the curve of figure 1 is representative of this material.

In figure 4, the calculated values have been obtained in the same manner as for figure 3 except that the column curve has been adjusted to be compatible with a compressive yield strength of 39,500 pounds per square inch. The buckling stress for the short specimens is still slightly higher than corresponding calculated values determined according to the Handbook formulas, even when complete fixity of the unloaded edges was assumed in the calculations.

It may be noticed in figure 4 that the points representing the test results indicate an increase in the buckling stress accompanying a decrease in length of the specimen, whereas the calculated curves in this region are horizontal lines. The formulas for equivalent slenderness ratio given in reference 1 are intended for design purposes and are based on the assumption that the loaded edges are simply supported. In such a case, the critical buckling stress for a plate supported or fixed along the unloaded edges is independent of the ratio of length to width, L/b, for values greater than 1 or 0.65, respectively. (See reference 6.) The L/b ratio for the shortest I-shaped specimen tested was 0.97.

Since the specimens were tested between flat heads, the loaded edges of the web may be considered as very nearly fixed. In such a case, the buckling strength is not independent of the length of the specimen, the K value for equation (4) increasing appreciably for L/b ratios less than about 3. (See reference 6.) In figure 5, the portions of the curve representing calculated values for the critical stress in local buckling have been obtained by determining equivalent slenderness ratios from equation (5), values of K corresponding to the L/b ratios of the individual specimens being used. Although the experimental points are still slightly higher than the calculated curves, there is good agreement between the shape of the calculated curves and that defined by the points representing test results.

The equivalent-slenderness-ratio method of handling buckling of flat plates is known to give conservative results for buckling in the plastic range of stresses. A more accurate means of calculating the critical stress for buckling of flat plates beyond the elastic range is to use for E in equation (4) some reduced modulus value, E_F, varying with the average stress and determined from a stress-strain diagram of the material. A theoretically determined value for E_F will depend on the assumptions made in the derivation. In the case of an axially loaded straight rectangular column, the effective modulus may be defined by the equation (reference 4, p. 159)

$$E_F = E_R = \frac{4EE_\sigma}{\left(\sqrt{E} + \sqrt{E_\sigma}\right)^2} \qquad (6)$$

where E_σ is the modulus of elasticity at stress, σ, (slope of stress-strain diagram at this stress).

In the buckling of flat plates subjected to edge compression in one direction, stresses occur in a direction normal to that of the applied force. The effective modulus value applicable in this case depends on what effect plastic action in the direction of the applied force has on the resistance of the material to stresses normal to this direction. (See reference 4, p. 384.) If it is assumed that the material remains isotropic, the effective modulus value is the E_R of equation (6). If it is assumed that plastic action in the direction of applied force does not affect the resistance at right angles, the effective modulus value becomes $\sqrt{EE_R}$. The actual behavior lies somewhere between these two extremes. An average value is sometimes used. The assumption of isotropic action is obviously on the conservative side. In figure 6, the solid portions of the calculated curves representing local buckling have been determined by using equation (4), replacing E by E_R as obtained by applying equation (6) to the stress-strain diagram of figure 1. The dotted curves representing local buckling are based on effective modulus values determined from the equation

$$E_F = \frac{E_R + \sqrt{EE_R}}{2} \tag{7}$$

The points representing test results fall between the curves for the two extreme conditions of restraint at the unloaded edges, that is, fixed and simply supported.

The relations between stress and effective modulus, for the stress-strain diagram of figure 1, are shown in figure 7. In the determination of the critical stress for buckling in the plastic range, E_F must be substituted for E in equation (4). The value of E_F used, however, must agree with the value of σ_{cr} determined from the equation. The equation can be solved directly if a curve is plotted showing the relation between the stress, σ, and the ratio, E_F/σ. (The direct determination of the critical stress when in the plastic range is accomplished in a slightly different manner in reference 7.) Since equation (4) can be written

$$\frac{E_F}{\sigma_{cr}} = (1 - \mu^2) \frac{\left(\frac{b}{t}\right)^2}{K} \tag{8}$$

the ratio, E_F/σ_{cr}, can be calculated, and a corresponding value of σ_{cr} can then be obtained from the curve. Such curves showing the relation between σ and E_F/σ have been plotted in figure 8 for the stress-modulus curve of figure 7.

CONCLUSIONS

From a study of the agreement between experimentally determined values of critical stress for buckling of the thin web of an I-beam in edge compression and corresponding values calculated in various manners (figs. 3 to 6), it is evident that the methods of calculation given on page 41 in reference 1 give values that would be satisfactory for design purposes. The assumption that the critical stress for the web is independent of the length of the specimen and the equivalent-slenderness-ratio method of handling buckling in the plastic range both give results that are slightly conservative. The agreement between experimental and calculated values of critical stress can be improved by considering the effect of the restraint at the loaded edges of the web and adopting a more accurate treatment for buckling beyond the elastic range.

Aluminum Research Laboratories,
Aluminum Company of America,
New Kensington, Pa., September 5, 1940.

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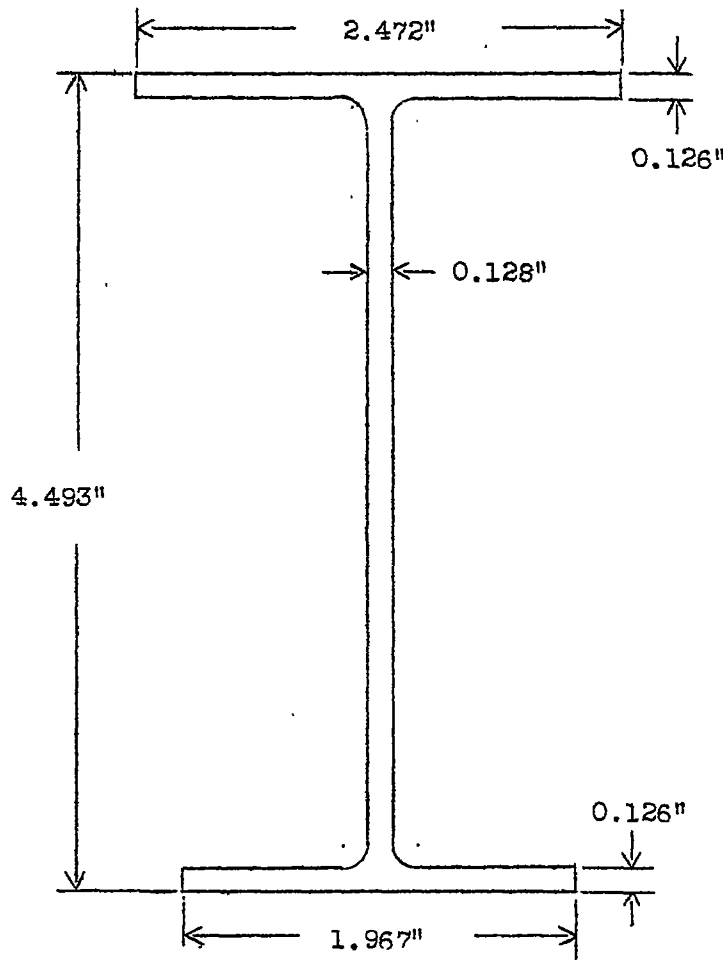
Table I.- Dimensions of specimens



Specimen no.	Length (in.)	Area (sq in.)
1	90.047	1.100
2	60.031	1.111
3	36.094	1.100
4	24.109	1.111
5	12.078	1.115
6	8.094	1.095
7	4.109	1.108
	Average	1.106

Average radius of gyration = 0.463 inch

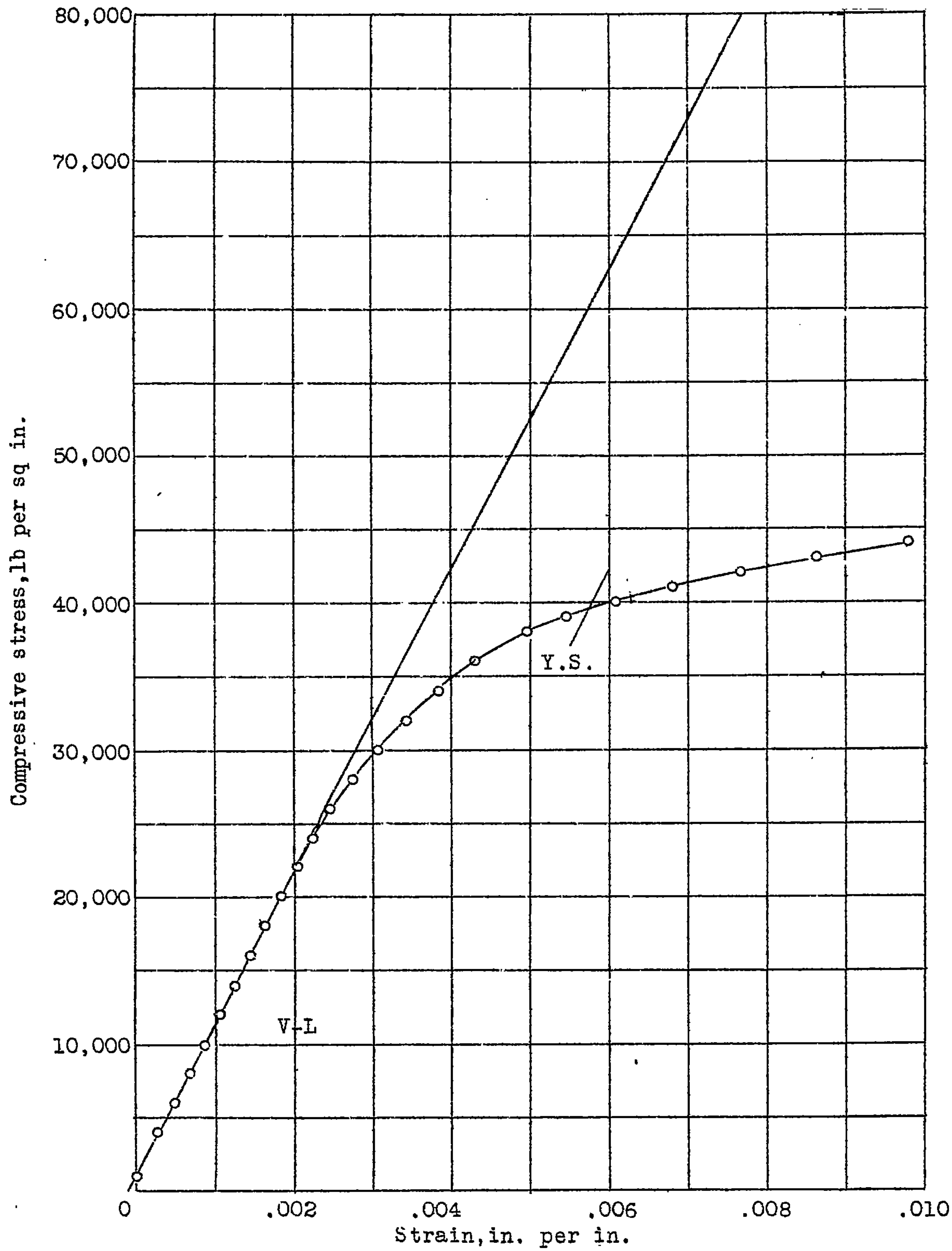


Figure 1.- Compressive stress-strain curve for 17S-T aluminum alloy extruded I-beam.

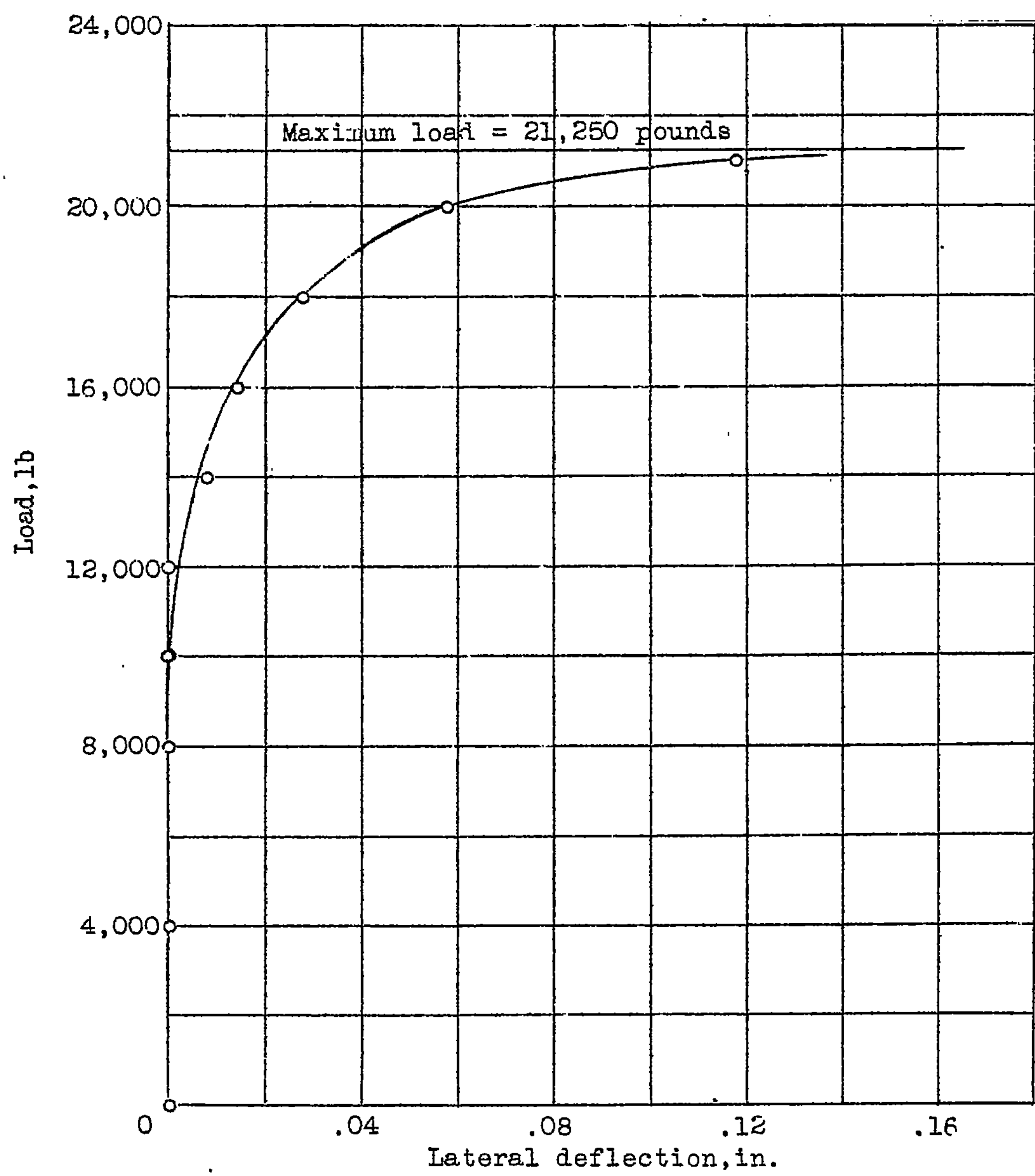


Figure 2.- Load-lateral deflection curve for specimen 2 of 17S-T aluminum alloy.

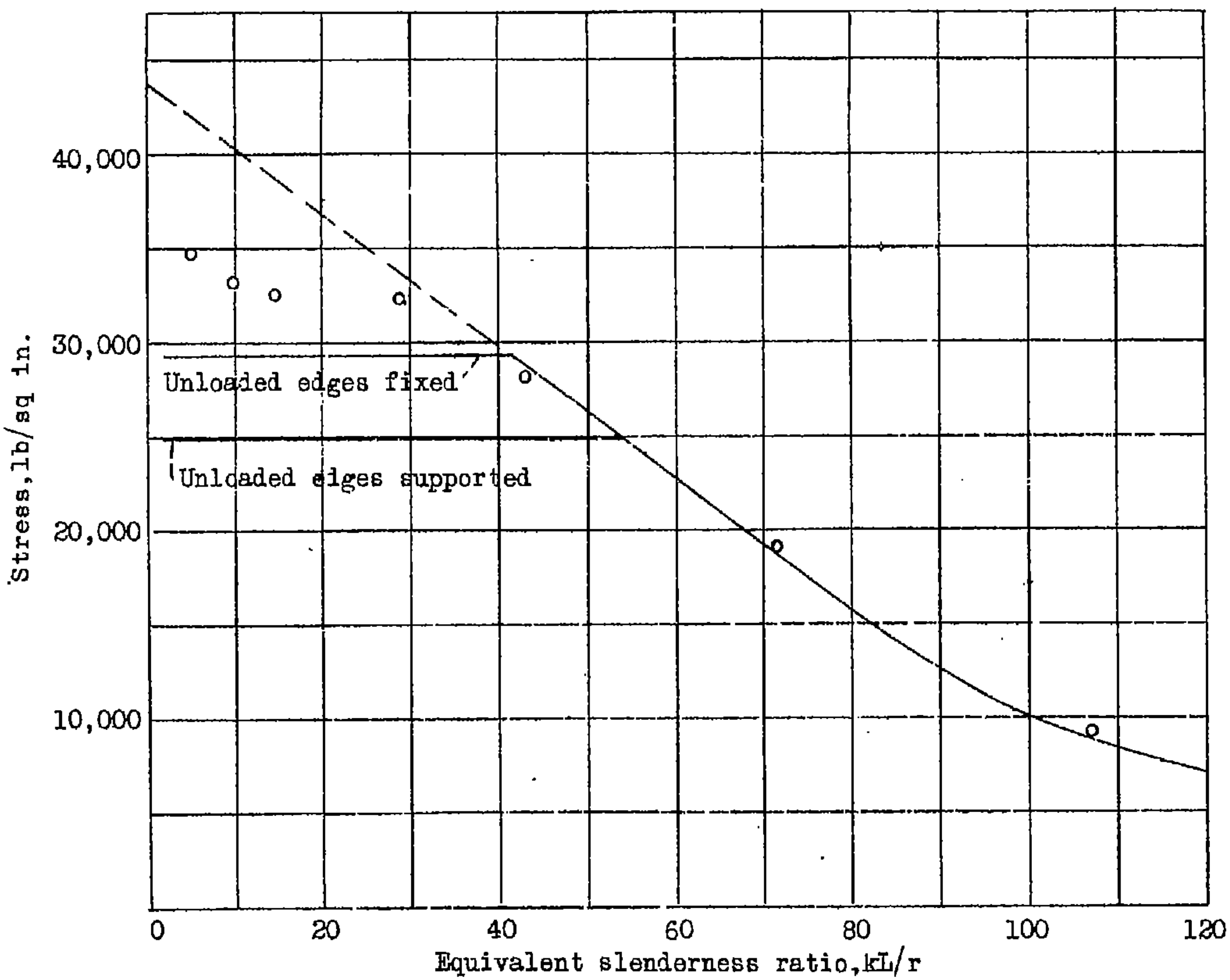


Figure 3.- Comparison between calculated and experimentally determined values of critical stress of 17S-T aluminum alloy. Values calculated according to the formulas on page 37 of reference 1 and typical properties of the material. Compressive yield strength, CYS, 37,000 pounds per square inch.

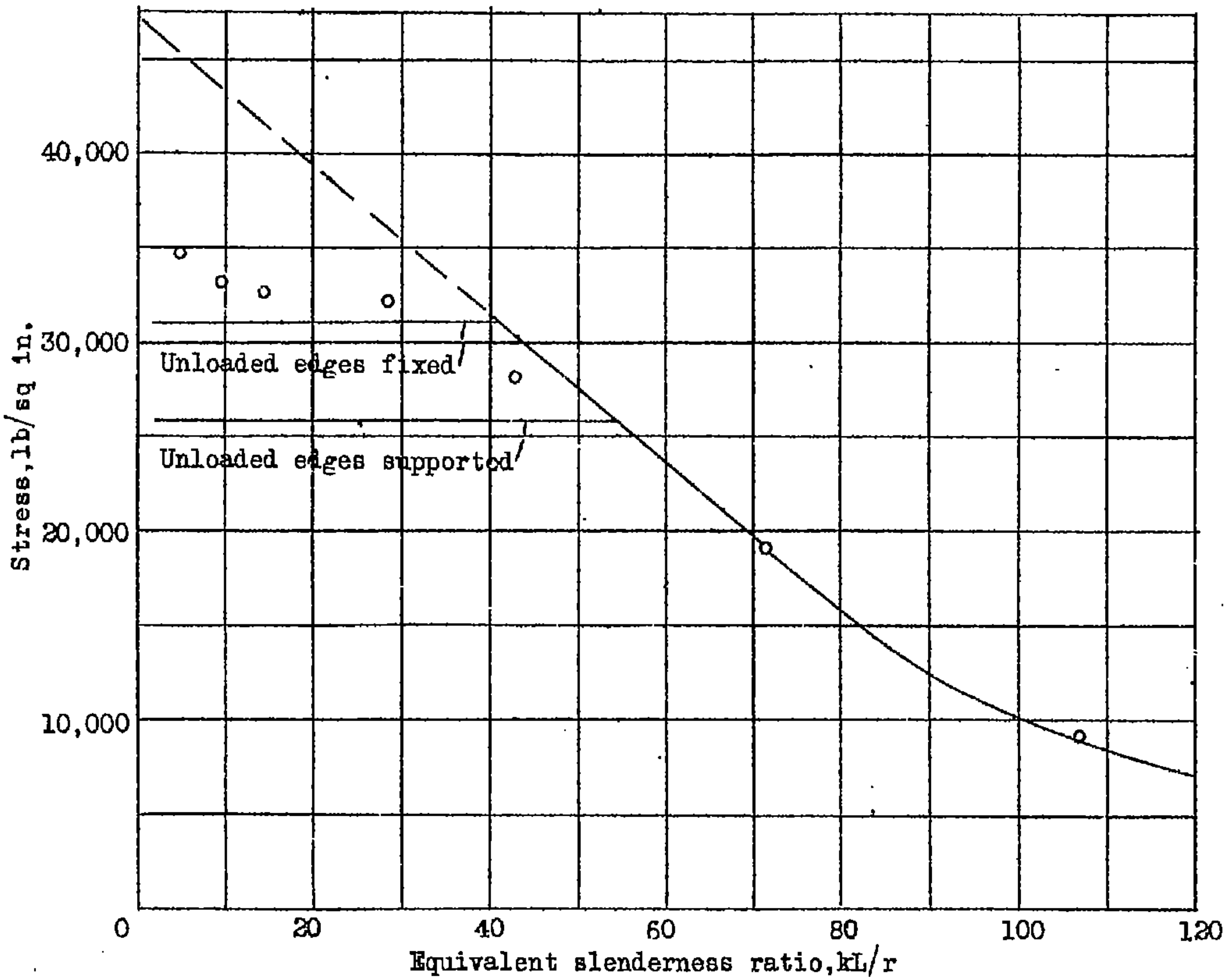


Figure 4.- Comparison between calculated and experimentally determined values of critical stress of 17S-T aluminum alloy. Values calculated according to the formulas on page 37 of reference 1. Value of compressive yield strength, CYS, determined from test as 39,500 pounds per square inch.

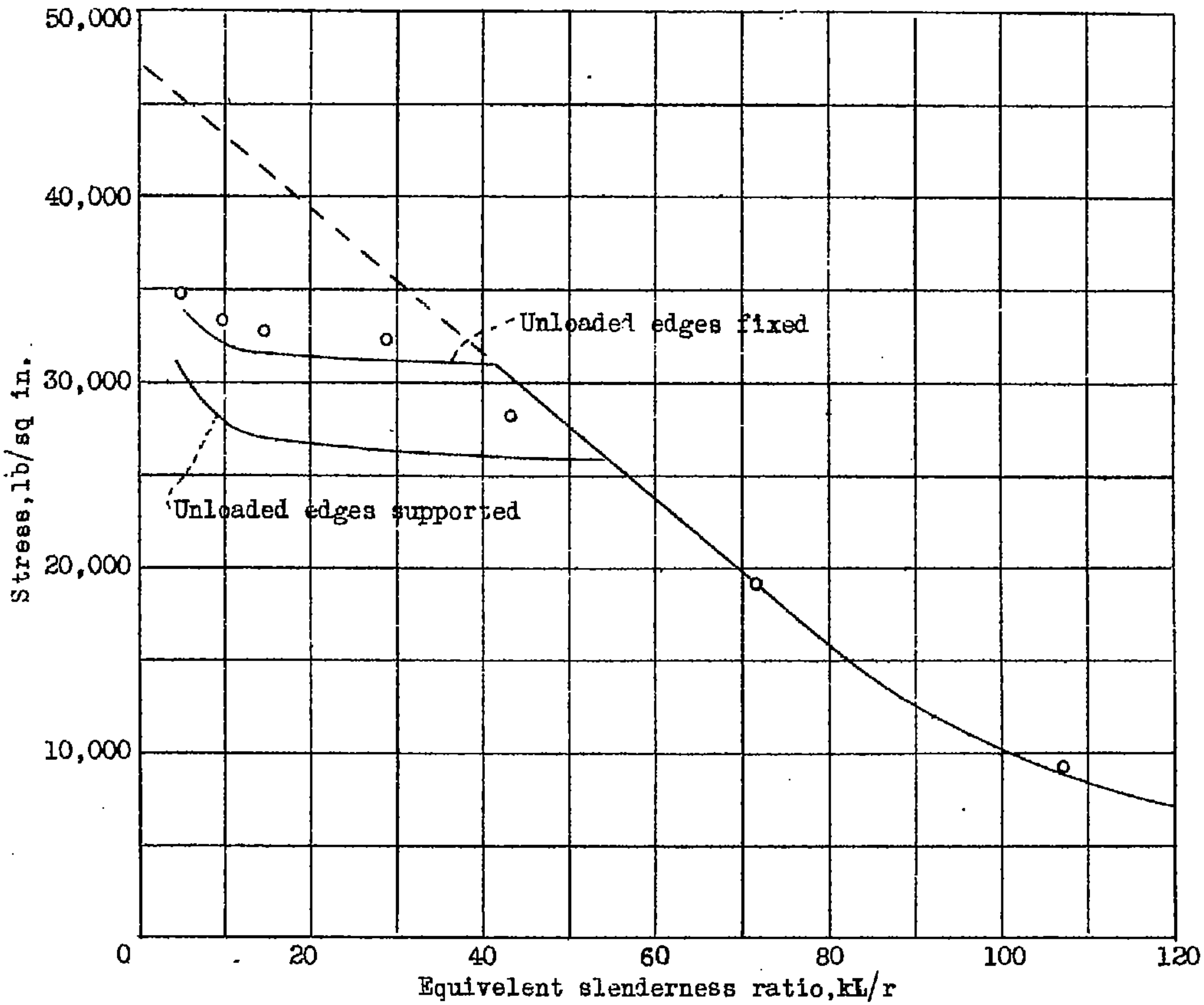


Figure 5.- Comparison between calculated and experimentally determined values of critical stress of 17S-T aluminum alloy. For local buckling, loaded edges assumed fixed; equivalent-slenderness-ratio method used for calculating critical stress.

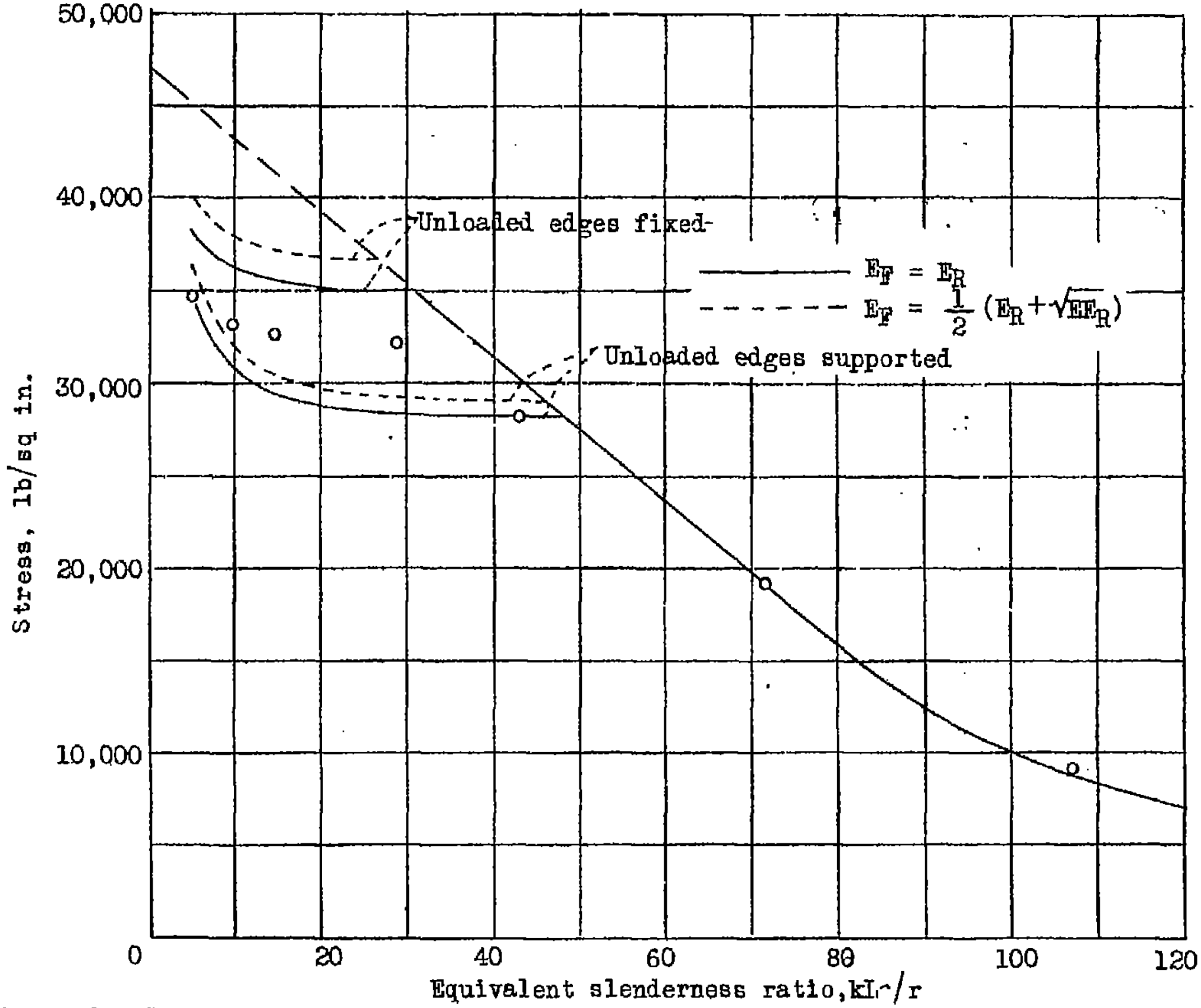


Figure 6.- Comparison between calculated and experimentally determined values of critical stress of 17S-T aluminum alloy. For local buckling, loaded edges assumed fixed; reduced modulus, E_F, method of reference 4 (p. 384) used for calculating critical stress.

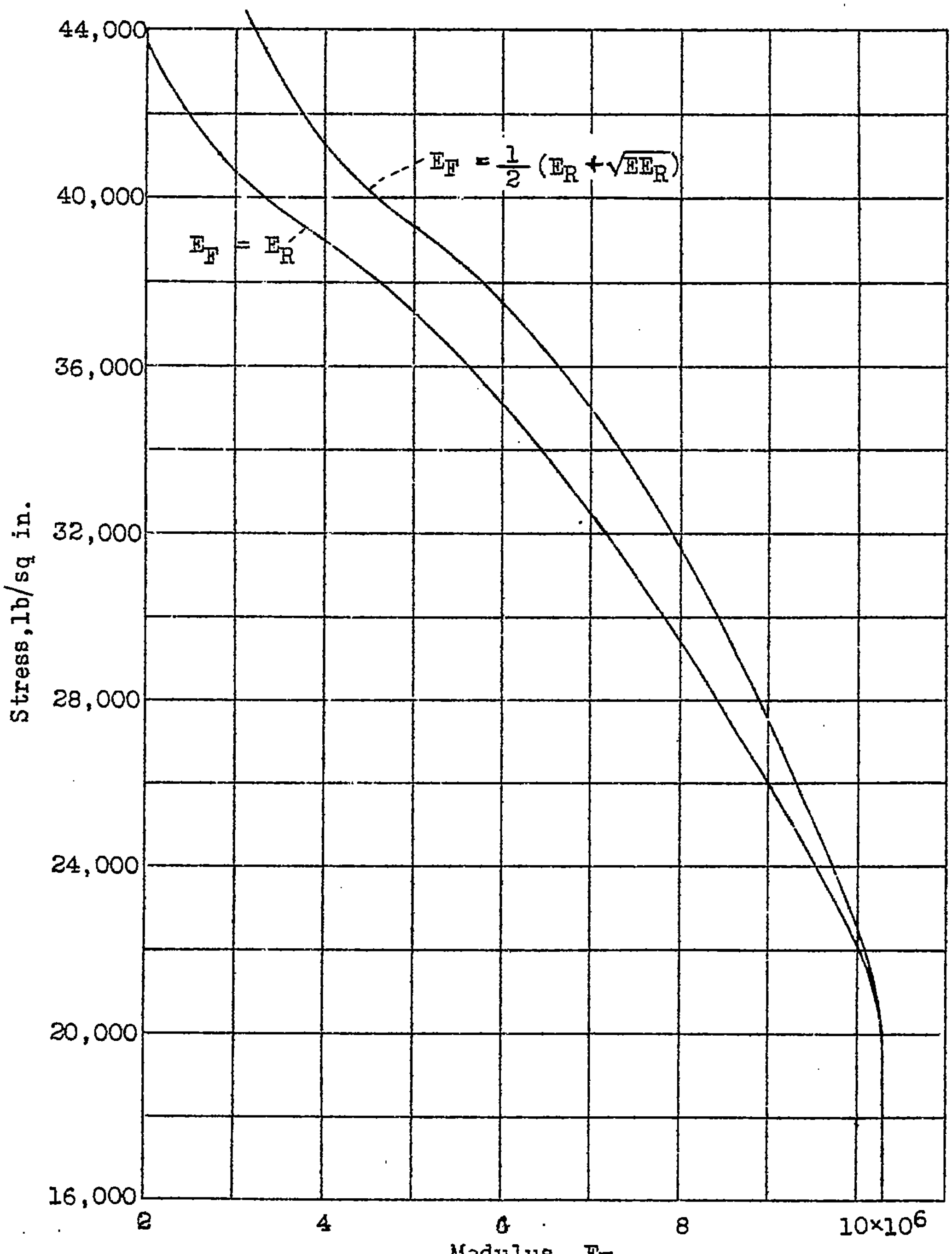


Figure 7.- Stress-modulus, E_F, curve for stress-strain diagram of figure 1. 17S-T aluminum alloy.

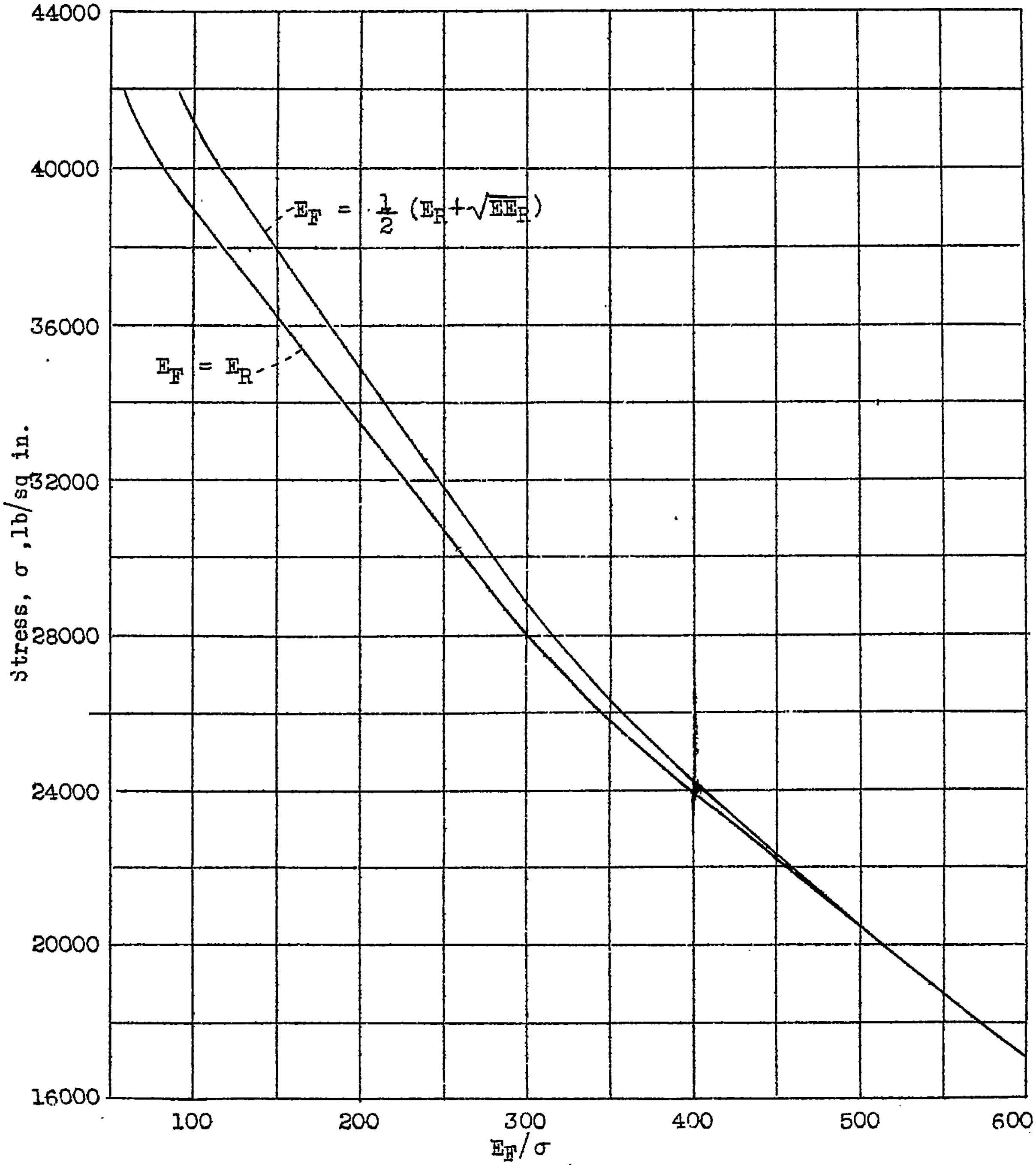


Figure 8.- Variation of σ with E_F/σ for stress-modulus curves of figure 7.